Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE
In Mechanics M4 (6680/01)

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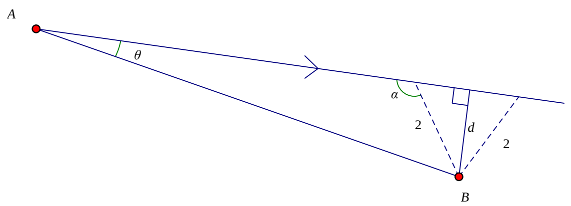
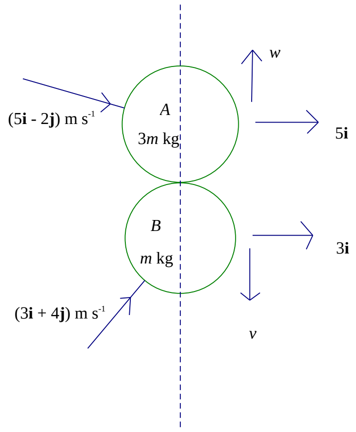
General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Q	Scheme	Marks	Notes
1a	Position vectors after t hours: $\begin{pmatrix}-2+4t\\1+2t\end{pmatrix}$ and $\begin{pmatrix}5-t\\-2+3t\end{pmatrix}$	M1	Use position vectors to find position of one ship relative to the other
	$\pm\begin{pmatrix}7-5t\\-3+t\end{pmatrix}$	A1	
	$d^2=(7-5t)^2+(-3+t)^2$ $=26t^2-76t+58$	M1	Correct method for magnitude
	Differentiate: $52t-76=0$	DM1	or complete the square
	$t=\dfrac{76}{52}=1.46\,\text{hrs},\ 13.28$	A1	
		(5)	
1a alt	Position vectors after t hours: $\begin{pmatrix}-2+4t\\1+2t\end{pmatrix}$ and $\begin{pmatrix}5-t\\-2+3t\end{pmatrix}$	M1	Use position vectors to find position of one ship relative to the other
	$\pm\begin{pmatrix}7-5t\\-3+t\end{pmatrix}$	A1	
	At closest point: $\begin{pmatrix}5\\-1\end{pmatrix}\bullet\begin{pmatrix}7-5t\\-3+t\end{pmatrix}=0$, $5(7-5t)-(-3+t)=0$	M1	Scalar product of relative velocity and relative position
	$35-25t+3-t=0$	DM1	Scalar product = 0
	$t=\dfrac{38}{26}=1.46\,\text{hrs},\ 13.28$	A1	
		(5)	
1a alt	Angle between initial positions of A and B and relative velocity $=\theta=\tan^{-1}\left(\dfrac{3}{7}\right)-\tan^{-1}\left(\dfrac{1}{5}\right)$		
	$=11.89°$	M1A1	
	$d=\sqrt{58}\cos\theta$	M1	
	Time taken $=\dfrac{d}{\sqrt{26}}$	DM1	
	$t=1.46\,\text{hrs},\ 13.28$	A1	
		(5)	

1b	Distance $\le 2: d^2 = 26t^2 - 76t + 58 \le 4$, $26t^2 - 76t + 54 \le 0$	M1	(condone equality)
	Time interval: $2\times\frac{\sqrt{76^2 - 4\times26\times54}}{52}\left(=2\frac{\sqrt{160}}{52}\right)$	M1	Difference between roots
	$=0.487$ hrs (29 mins)	A1	0.49 or better
		(3)	
1balt	$\frac{\sin\alpha}{\sqrt{58}} = \frac{\sin 11.89}{2} \Rightarrow \alpha = 51.7°, 128.3°$ $d_1 = 8.695,\ \ d_2 = 6.214$	M1	Find at least one distance
	$t = \frac{d}{\sqrt{26}} \Rightarrow t_1 = 1.705,\ t_2 = 1.219$	M1	
	Interval $=0.486$ hrs (29 mins)	A1	0.49 or better
		(3)	
		[8]	

			
2a	For *A*, component perpendicular to loc = 5	B1	
	For *B*, component perpendicular to loc = 3	B1	
	$\frac{1}{2}m\times 25\times\frac{85}{100}=\frac{1}{2}m\left(3^3+v^2\right)$	M1	Equation for kinetic energy of *B* For their "3"
	$\frac{85}{4}=9+v^2\,,\quad v^2=\frac{49}{4}$	A1	
	$-6m+4m=3mw-mv$ $\left(=3mw-3.5m\right)$	M1	CLM parallel to loc. No missing/additional terms Condone sign error(s)
		A1ft	Correct unsimplified equation for CLM (with their values if substituted)
	$w=0.5$		
	Select correct root and state velocities:	DM1	
	$\mathbf{v}_B=\left(3\mathbf{i}-3.5\mathbf{j}\right)$ (m s^{-1})	A1	One correct
	$\mathbf{v}_A=\left(5\mathbf{i}+0.5\mathbf{j}\right)$ (m s^{-1})	A1	Both correct
		(9)	
2b	$v+w=e\left(2+4\right)$	M1	Impact law parallel to loc. Used the right way round. Condone sign error(s)
	$0.5+3.5=6e$	A1ft	Correct unsimplified or with their values
	$e=\frac{2}{3}$	A1	
		(3)	
		[12]	

3a		M1	Differential equation in v and x No additional/missing terms. Condone sign error(s)
	$75v\dfrac{dv}{dx}=150-3v^2$	A1	
	$\displaystyle\int\frac{75v}{150-3v^2}\,dv=\int 1\,dx$	M1	Separate variables
	$x=\left[-\dfrac{25}{2}\ln\left(50-v^2\right)\right]_0^v$	DM1	Integrate and use limits
	$=-\dfrac{25}{2}\ln\left(\dfrac{50-v^2}{50}\right)$	A1	
	$-\dfrac{2x}{25}=\ln\left(1-\dfrac{v^2}{50}\right), v^2=50\left(1-e^{\frac{-2x}{25}}\right)$	DM1	Change the subject to v or v^2
	$v=\sqrt{50\left(1-e^{\frac{-2x}{25}}\right)}$	A1	
		(7)	
3a alt	$\dfrac{dv^2}{dx}+\dfrac{2}{25}v^2=4$	M1A1	
	Integrating factor: $e^{\frac{2}{25}x}$	M1	
	$v^2e^{\frac{2}{25}x}=\int 4e^{\frac{2}{25}x}dx=50e^{\frac{2}{25}x}\ (+C)$ $v=0, x=0\Rightarrow C=-50$	M1A1	Integrate and use limits
	$v^2=50-50e^{-\frac{2}{25}x}$	DM1	Change the subject to v or v^2
	$v=\sqrt{50\left(1-e^{\frac{-2x}{25}}\right)}$	A1	
		(7)	

3b	$75\dfrac{dv}{dt}=150-3v^2$	M1	Differential equation in v and t No additional/missing terms. Condone sign error(s)
	$\int\dfrac{75}{150-3v^2}\,dv=\int 1\,dt$	M1	Separate variables and integrate
	$t=\int\dfrac{25}{50-v^2}\,dv$ $=\dfrac{25}{2\sqrt{50}}\int\dfrac{1}{\sqrt{50}+v}+\dfrac{1}{\sqrt{50}-v}\,dv$		
	$=\dfrac{25}{2\sqrt{50}}\left(\ln\left(\sqrt{50}+v\right)-\ln\left(\sqrt{50}-v\right)\right)$	A1	With or without constant of integration Or $\dfrac{25}{\sqrt{50}}\operatorname{arc\,tanh}\dfrac{v}{\sqrt{50}}$
	$t=\dfrac{25}{2\sqrt{50}}\ln\left(\dfrac{\sqrt{50}+v}{\sqrt{50}-v}\right)-\dfrac{25}{2\sqrt{50}}\ln\left(\dfrac{\sqrt{50}}{\sqrt{50}}\right)$	DM1	Use limits 0 and v
	$=\dfrac{25}{2\sqrt{50}}\ln\left(\dfrac{\sqrt{50}+v}{\sqrt{50}-v}\right)$	A1	$\left(=\dfrac{5\sqrt{2}}{4}\ln\left(\dfrac{\sqrt{50}+v}{\sqrt{50}-v}\right)\right)$ or equivalent
		(5)	
		[12]	

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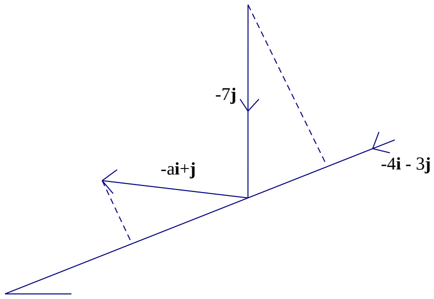
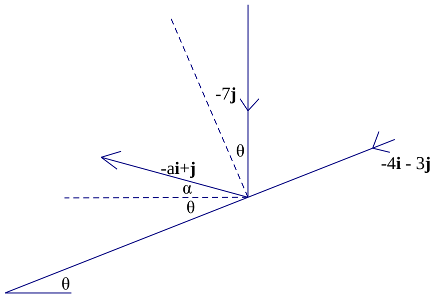
4a			
	Components parallel to the plane unchanged: $\left(\begin{pmatrix}0\\-7\end{pmatrix}\bullet\frac{1}{5}\begin{pmatrix}-4\\-3\end{pmatrix}=\begin{pmatrix}-a\\1\end{pmatrix}\bullet\frac{1}{5}\begin{pmatrix}-4\\-3\end{pmatrix}\right)$ $\Rightarrow 21=4a-3$	M1	Use of scalar product. Do not need to see $\frac{1}{5}$
	$a=6$	A1	***Given Answer***
		(2)	
4a alt			
	Components parallel to the plane: $7\sin\theta=v\cos(\theta+\alpha)$		$\theta+\alpha=46.3...°$
	$7\sin\theta=v(\cos\theta\cos\alpha-\sin\theta\sin\alpha)$, $\Rightarrow 7\tan\theta=a-\tan\theta$	M1	Equate components and form an equation in a and θ
	$8\tan\theta=a=6$	A1	
		(2)	
4b	Component of -7**j** parallel to the plane $=\frac{21}{\lvert 4\mathbf{i}+3\mathbf{j}\rvert}$	M1	Scalar product of -7**j** and unit vector parallel to plane
	$=4.2$	A1	
	$\sqrt{49-4.2^2}=\sqrt{31.36}$	M1	Use Pythagoras to find components perpendicular to the plane
	$\sqrt{36+1-4.2^2}=\sqrt{19.36}$	A1	Both correct
	$\sqrt{19.36}=e\times\sqrt{31.36}$	DM1	Use if impact law Dependent on preceding M mark
	$e=0.786$	A1	
		(6)	

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Alt4 b	Component of -7**j** perpendicular to the plane $=\frac{1}{5}\begin{pmatrix}0\\-7\end{pmatrix}\bullet\begin{pmatrix}-3\\4\end{pmatrix}$	M1A1	
	Component of $-a\mathbf{i}+\mathbf{j}$ perpendicular to the plane $=\frac{1}{5}\begin{pmatrix}-6\\1\end{pmatrix}\bullet\begin{pmatrix}-3\\4\end{pmatrix}$	M1A1	
	Impact law: $e=\dfrac{\frac{1}{5}\times 22}{\frac{1}{5}\times 28}=\frac{22}{28}=\frac{11}{14}(=0.786)$	DM1 A1	
		(6)	
Alt 4b	Components perpendicular to the plane: $e\times 7\cos\theta = v\sin(\theta+\alpha)$	M1	
	$e\times 7\cos\theta = v(\sin\theta\cos\alpha+\cos\theta\sin\alpha)$	A1	$\theta+\alpha=46.3...°$, $v=\sqrt{37}$
	Substitute for α: $7e=6\tan\theta+1$	M1A1	
	Solve for e: $7e=6\times\frac{3}{4}+1=\frac{11}{2}$, $e=\frac{11}{14}$	DM1 A1	
		(6)	
Alt 4b	Components perpendicular to the plane: $e\times 7\cos\theta = v\sin(\theta+\alpha)$	M1	
	$e\times 7\cos\theta = v(\sin\theta\cos\alpha+\cos\theta\sin\alpha)$	A1	
	Divide and substitute for α: $e\cot\theta=\tan(\theta+\alpha)=\dfrac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha}$	M1	
	$=\dfrac{\frac{3}{4}+\frac{1}{6}}{1-\frac{3}{4\times 6}}=\dfrac{3\times 6+4}{4\times 6-3}$	A1	
	Solve for e: $e=\frac{22}{21}\times\frac{3}{4}=\frac{11}{14}$	DM1 A1	
		(6)	

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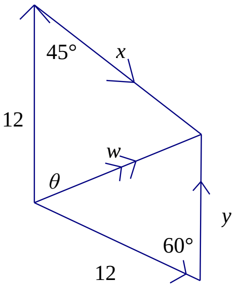
Q	Scheme	Marks	Notes
Alt 4b	Parallel: $7\sin\theta = \sqrt{37}\cos(\theta+\alpha)$ Perpendicular: $e7\cos\theta = \sqrt{37}\sin(\theta+\alpha)$	M1A1	Pair of equations
	$49\sin^2\theta = 37\cos^2(\theta+\alpha)$ $\Rightarrow \sin^2(\theta+\alpha) = 1 - \frac{49}{37}\sin^2\theta$	M1	Square and substitute to eliminate $\theta+\alpha$
	$49e^2\cos^2\theta = 37\sin^2(\theta+\alpha) = 37 - 49\sin^2\theta$	A1	
	$e^2 = \dfrac{37 - 49\times\frac{9}{25}}{49\times\frac{16}{25}} = \dfrac{121}{196}, \quad e = \dfrac{11}{14}$	M1A1	Substitute for θ to obtain e.
		(6)	
		[8]	

Q	Scheme	Marks	Notes
5		B1 B1	One correct triangle Two triangles combined using their common *w*. (seen or implied)
	$12\cos 30° = x\cos 45°$	M1	Horizontal components equal
	$x = 12\sqrt{\frac{3}{2}}\left(= 6\sqrt{6} = 14.6......\right)$	A1	
	$w^2 = 12^2 + x^2 - 2\times 12 x\cos 45°$	M1	Cosine rule
	$w = 10.5\,(\text{km h}^{-1})$	A1	
	EITHER: Sine Rule: $\frac{\sin\theta}{x} = \frac{\sin 45°}{w}$ $\left(\frac{\sin\theta}{12} = \frac{\sin 60°}{w}\right)$	M1	
	$\theta = 81.2°$	A1	
	Direction $261°$	A1	
	OR: $\updownarrow\; w\cos\theta = 12 - 12\cos 30°$ $\leftrightarrow\; w\sin\theta = 12\cos 30°$		
	$\Rightarrow \tan\theta = \frac{\cos 30°}{1-\cos 30°}$,	(M1)	
	$\theta = 81.2°$	(A1)	
	Direction $261°$	(A1)	
		(9)	
5 alt	$\mathbf{w} = \begin{pmatrix} x\cos 45° \\ 12 - x\cos 45° \end{pmatrix}$	B1	**w** expressed as a vector
	$\mathbf{w} = \begin{pmatrix} 12\cos 30° \\ -12\sin 30° + y \end{pmatrix}$	B1	Second expression of **w** as a vector
	$12\cos 30° = x\cos 45°$	M1	Horizontal components equal
	$x\cos 45° = 6\sqrt{3}$	A1	Or $x = 6\sqrt{6}$
	$\lvert\mathbf{w}\rvert^2 = 3\times 36 + \left(12 - 6\sqrt{3}\right)^2$	M1	Use of Pythagoras
	$\lvert\mathbf{w}\rvert = 10.5$ (km h^{-1})	A1	
	$\tan\theta = \frac{6\sqrt{3}}{12 - 6\sqrt{3}}$	M1	Correct method for direction of **w**
	$\theta = 81.2°$	A1	
	Direction $261°$	A1	
		(9)	
		[9]	

Q	Scheme	Marks	Notes
6a	$\frac{7e}{0.8}=0.2g$	B1	$T=mg$ at equilibrium (e = 0.224)
	$0.2\ddot{x}=0.2g-\frac{7(x+e)}{0.8}-2v$	M1	Equation of motion No missing/additional terms Condone sign error(s) Allow with m and l
		A1	At most one error m and l substituted
	$\left(0.2\ddot{x}=-\frac{7x}{0.8}-2\dot{x}\right)$	A1	Correct unsimplified equation m and l substituted
	$\ddot{x}+10\dot{x}+43.75x=0$	A1	***given answer***
		(5)	
6b	AE: $m^2+10m+43.75=0$, $m=-5\pm\mathrm{i}\frac{5\sqrt{3}}{2}$	M1	Form and solve AE
	$x=e^{-5t}\left(A\cos\frac{5\sqrt{3}}{2}t+B\sin\frac{5\sqrt{3}}{2}t\right)$	A1	
	$t=0,x=0.2\quad\Rightarrow A=0.2$	B1	
	$\dot{x}=-5e^{-5t}\left(A\cos\omega t+B\sin\omega t\right)$ $+e^{-5t}\left(-A\omega\sin\omega t+B\omega\cos\omega t\right)$	M1	
	$0=-5A+B\omega=-1+B\omega,\ \ B=\frac{2}{5\sqrt{3}}$	M1	Use of $t=0,\dot{x}=0$ to find B
	$x=\frac{e^{-5t}}{5}\left(\cos\frac{5\sqrt{3}}{2}t+\frac{2}{\sqrt{3}}\sin\frac{5\sqrt{3}}{2}t\right)$	A1	
		(6)	
6c	instantaneous rest when $t=\frac{\pi}{\omega}$ (s)	M1	Use of periodic time $=\frac{2\pi}{\omega}$
	$=\frac{2\pi}{5\sqrt{3}}=0.73$	A1	
		(2)	
6c alt	$\dot{x}=-5e^{-5t}\left(\frac{1}{5}\cos\omega t+\frac{2}{5\sqrt{3}}\sin\omega t\right)$ $+e^{-5t}\left(-\frac{1}{5}\omega\sin\omega t+\frac{2}{5\sqrt{3}}\omega\cos\omega t\right)$		
	$=\sqrt{3}e^{-5t}\sin\frac{5\sqrt{3}}{2}t=0$	M1	Solve $\dot{x}=0$
	$\Rightarrow\frac{5\sqrt{3}}{2}t=\pi$, $\ t=\frac{2\pi}{5\sqrt{3}}=0.73\,\mathrm{s}$	A1	
		(2)	
		[13]	

Q	Scheme	Marks	Notes
7a	Relative to the fixed point A, PE of mass at C $=-3mg\times 4a\cos\theta$	B1	
	PE of rods $=-2mg\times a\cos\theta-2mg\times 3a\cos\theta$	B1	
	Extension in the spring $=4a\cos\theta-2a$	B1	
	$\dfrac{7mg\left(4a\cos\theta-2a\right)^2}{4a}-20mga\cos\theta$	M1	Total PE
	$=7mga\left(4\cos^2\theta-4\cos\theta+1\right)$ $-20mga\cos\theta$		
	$=28mga\cos^2\theta-48mga\cos\theta$ + constant	A1	***given answer***
		(5)	
7b	$\dfrac{\mathrm{d}V}{\mathrm{d}\theta}=$	M1	Differentiate
	$=-56mga\cos\theta\sin\theta+48mga\sin\theta$	A1	
	$8\sin\theta\left(-7\cos\theta+6\right)=0$	M1	$\dfrac{\mathrm{d}V}{\mathrm{d}\theta}=0$ and solve for θ
	$\Rightarrow\theta=\cos^{-1}\dfrac{6}{7}\left(31°, 0.54\mathrm{r}\right)$	A1	
		(4)	
7c		M1	Differentiate to obtain second derivative
	$\dfrac{\mathrm{d}^2V}{\mathrm{d}\theta^2}=-56mga\left(\cos^2\theta-\sin^2\theta\right)$ $+48mga\cos\theta$	A1	
	$=-56mga\left(2\times\dfrac{36}{49}-1\right)+48mga\times\dfrac{6}{7}$	DM1	Find value of second derivative when $\theta=\cos^{-1}\dfrac{6}{7}$ Dependent on preceding M1
	$=\dfrac{728}{49}mga>0$, stable	A1	(14.8...mga)
		(4)	
		[13]	